

EEE204

BASIC LAWS

CIRCUIT ELEMENTS

- Elements of an electric circuit can be interconnected in several ways.
- To understand these interconnections, a knowledge of the basic concepts of network topology is needed.
- Placement of elements in the network and the geometric configuration of the network are the major aspects of network topology

Branch, Node and loop

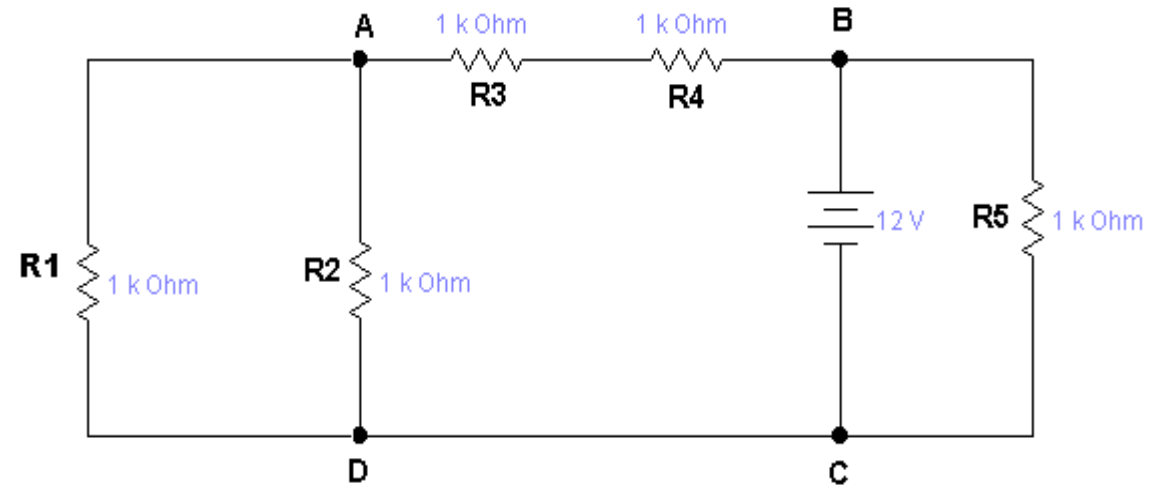
- A **branch** represents a single element such a voltage source or a resistor.
- It represents any two-terminal element
- A **node** is the point of connection between two or more branches
- If a short circuit connects two nodes, the two nodes constitute a single node
- A **loop** is any closed path in a circuit
- For a path to be closed, the path must start a node, pass through other nodes, and returns to the starting nodes without passing through any node more than once

Independent loop

- A loop is said to be independent if it contains at least one branch which is not a part of any other independent loop
- Independent loops yield independent sets of equations.
- Note:
 - A network that has b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:
 - $$b = l + n - 1$$

Series Connection and Parallel Connection

- Two or more elements are in series if they exclusively share a single node.
 - Series elements carry the same current
- Two or more elements are in parallel if they are connected to the same two nodes.
 - Parallel elements have the same voltage across them

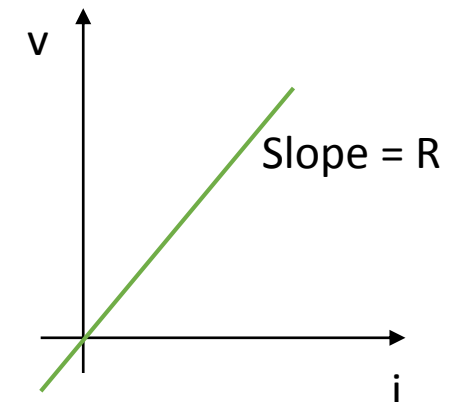
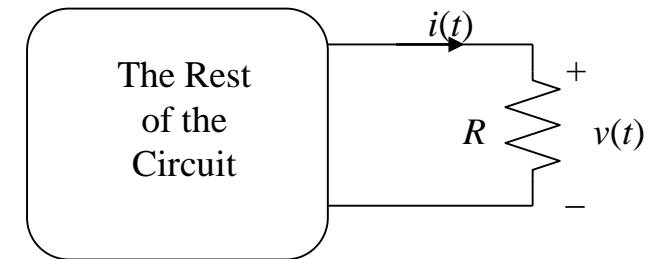


Series and Parallel.....

- R1 and R2 are connected to the same set of nodes (A, D)
- Consequently, R1 and R2 have the same voltage across them.
- R1 and R2 are said therefore to be in parallel
- R3 and R4 are connected together and as such, they exclusively share a single node
- Consequently, the same current flow through them
- R3 and R4 are said to be connected in series
- Also, the 12V source and R5 share the same set of nodes (B, C), therefore the same voltage is observed across them. This voltage has the value of 12V

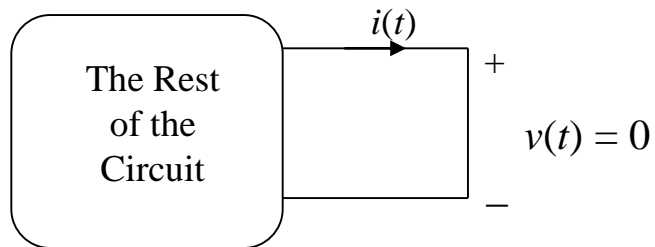
Ohm's Law

- **Ohm's law** states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor
- $v \propto i$
- Where the constant of proportionality is resistance R
- Mathematically,
 - $v = iR$ (1)
 - $i = \frac{v}{R}$ (2)



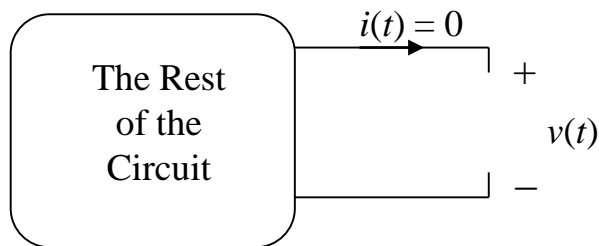
Short Circuit

- A short circuit is a circuit element with resistance approaching zero
- When $R = 0$, the element is seen as a perfect conductor and as such,
 - $i = \frac{v}{R} = \frac{v}{0} = \infty$
 - $v = iR = 0$



Open Circuit

- An open circuit is an element with an infinite resistance
- When $R = \infty$, the element is seen as a perfect insulator and as such,
 - $i = \frac{v}{R} = \frac{v}{\infty} = 0$



Current Direction

- To apply Ohm's law, the direction of current and the polarity of voltage must conform with passive sign convention.
- This implies that current flows from higher potential to a lower potential in order for $v = iR$.
- On the other hand, if current flows from a lower potential to a higher potential, $v = -iR$

Example 1: Given a 60W bulb, if the voltage across it is 230V, calculate:

(i) The resistance

(ii) The current flowing through it

• Solution

$$\bullet P = IV = \frac{V}{R} \times V = \frac{V^2}{R}$$

$$\bullet \therefore R = \frac{V^2}{P} = \frac{230^2}{60} = 881\Omega$$

$$\bullet I = \frac{V}{R} = \frac{230}{881} = 0.261A$$

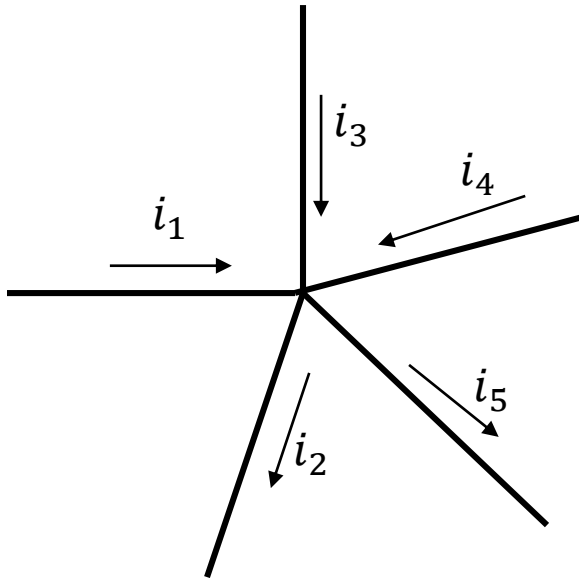
Kirchhoff's laws

- Introduced in 1847 by a German Physicist Gustav Robert Kirchhoff
- These laws are the Kirchhoff's Current law (KCL) and Kirchhoff's Voltage law (KVL)

Kirchhoff's Current Law (KCL)

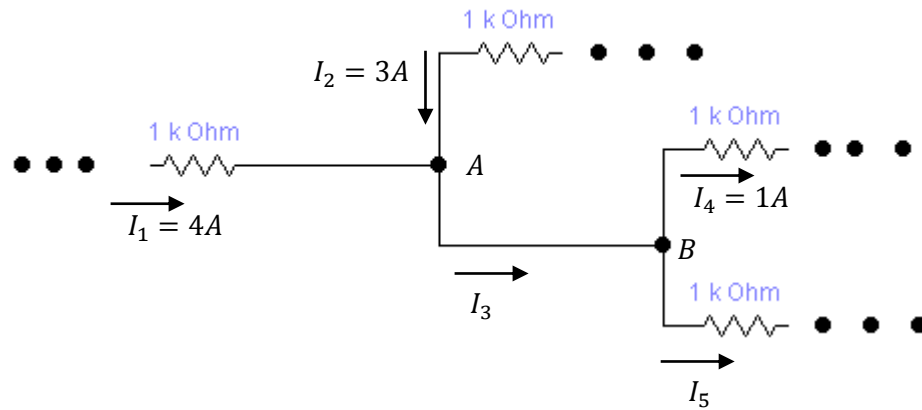
- The algebraic sum of currents entering a node (or a closed boundary) is zero
- Sum of currents entering node is equal to sum of currents leaving node
- Mathematically,
- $\sum_{n=1}^N i_n = 0 \dots \dots \dots (3)$
- Where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node
- This KCL is based on the law of conservation of charge

KCL Illustration



$$\sum_{n=1}^N i_n = 0$$
$$\therefore i_1 + i_3 + i_4 = i_2 + i_5$$
$$\therefore i_1 - i_2 + i_3 + i_4 - i_5 = 0$$

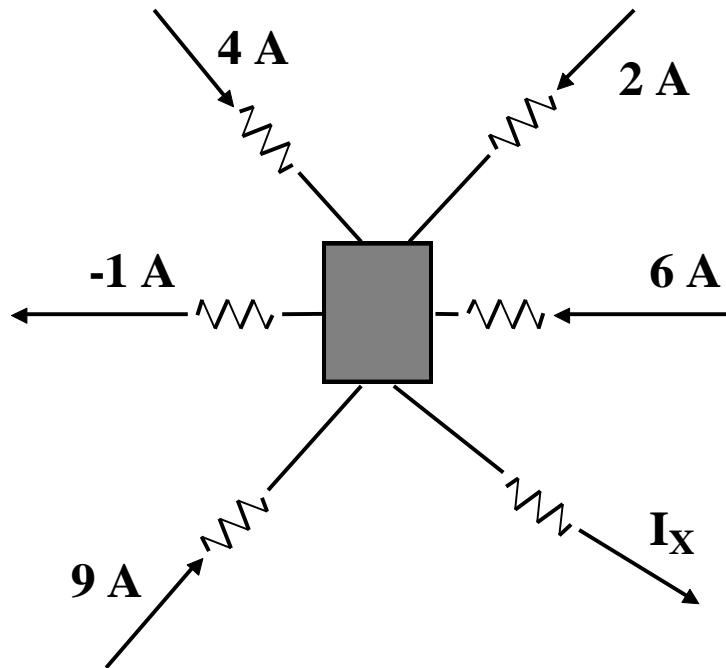
Example 2: Determine the values of I_3 and I_5



• SOLUTION

- For node A:
- *Sum of currents entering = Sum of currents leaving*
- $I_1 + I_2 = I_3$
- $\therefore I_3 = 4A + 3A = 7A$
- For node B:
- $I_5 + I_4 = I_3$
- $\therefore I_5 = 7A - 1A = 6A$

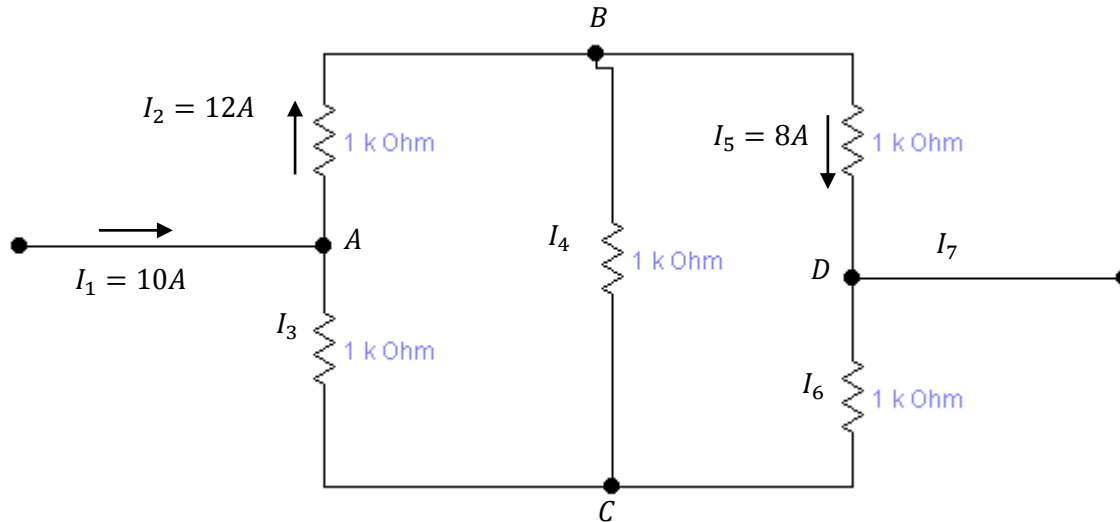
Example 3 : Find the current I_x .



- SOLUTION

- *Sum of currents entering the node = Sum of currents leaving the node*
- $\therefore 4 + 2 + 6 + 9 = -1 + I_x$
- $\Rightarrow I_x = 21 + 1 = 22A$

Example 4: Find the magnitude and directions of I_3, I_4, I_6 and I_7



- SOLUTION:

- Node A:

- $I_1 + I_2 + I_3 = 0$
- $\therefore 10 - 12 + I_3 = 0$
- $\therefore I_3 = 12 - 10 = 2A$
- Since I_3 is positive, it indicates that the current flows into node A

- Node B:

- $I_2 + I_4 + I_5 = 0$
- $\therefore 12 + I_4 - 8 = 0$
- $\therefore I_4 = -4A$
- (this implies that it flows out from node B)

- Node C:

- $I_3 + I_4 + I_6 = 0$
- $\therefore -2 + 4 + I_6 = 0$
- $\therefore I_6 = -2A$ (leaves node C)

- Node D:

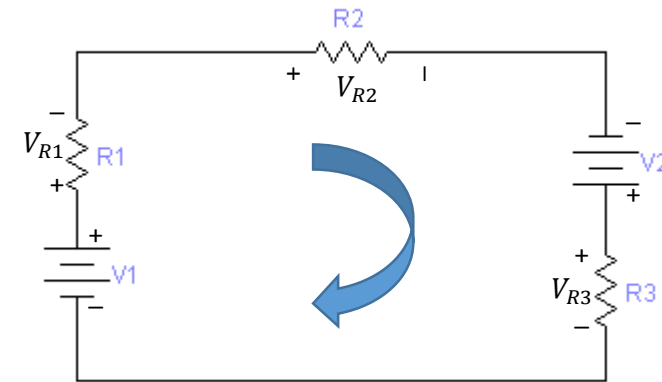
- $I_5 + I_6 + I_7 = 0$
- $\therefore 8 + 2 + I_7 = 0$
- $\therefore I_7 = -10A$ (leaves node D)

Kirchhoff's Voltage Law (KVL)

- The algebraic sum of all voltages around a closed path (or loop) is zero
- Mathematically,
 - $\sum_{m=1}^M v_m = 0 \dots \dots \dots (4)$
 - Where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the m th voltage
- KVL can be applied in two ways:
 - By taking a clockwise trip round the loop
 - By taking an anti-clockwise trip round the loop

KVL Contd

- KVL is based on the law of conservation of energy.
 - Sum of voltage drops = Sum of voltage rises
- Voltage Polarity Convention
- A sign on each voltage is the polarity of the terminal encountered first as we travel around the loop
 - If a voltage encountered has first the +ve terminal, then the voltage is given a +ve sign
 - If a voltage encountered has first the -ve terminal, then the voltage is given a -ve sign



- Starting from v_1
- $-V_1 + V_{R1} + V_{R2} - V_2 + V_{R3} = 0$
- $\therefore \underbrace{V_1 + V_2}_{\text{sum of voltage rises}} = \underbrace{V_{R1} + V_{R2} + V_{R3}}_{\text{sum of voltage drops}}$

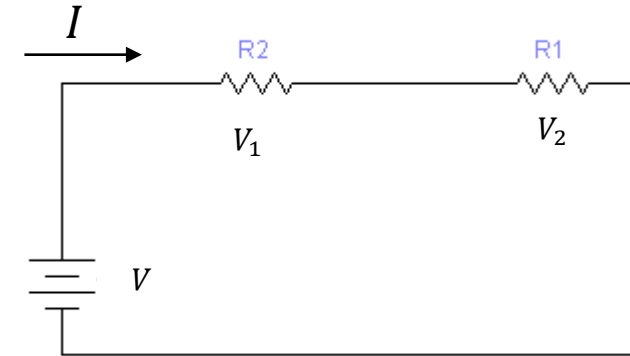
Example 5: Find V_1 and V_2 in the following circuit



- To find V_1 and V_2 , we apply both Ohm's law and KVL:
- $V_1 = 6I, V_2 = 3I$
- Applying KVL:
- $-12 + V_1 - 8 + V_2 = 0$
- $V_1 + V_2 = 20$
- $6I + 3I = 20$
- $\therefore I = \frac{20}{9} = 2.222A$
- $\therefore V_1 = 6 \times 2.222 = 13.33V$
- And $V_2 = 3 \times 2.222 = 6.67V$

SERIES Resistors and Voltage Division

- From KVL: $V - V_1 - V_2 = 0$
- But from Ohm's Law, $V_1 = IR_1$ and $V_2 = IR_2$
- $\therefore V - IR_1 - IR_2 = 0$
- $\Rightarrow V = I(R_1 + R_2)$
- $\therefore \frac{V}{I} = (R_1 + R_2) = R$
- This implies that when two resistors R_1 and R_2 are connected in series, they can be replaced by an equivalent resistor R_{eq} where:
 - $R_{eq} = R_1 + R_2 \dots \dots \dots (5)$
- For more than two resistors:
 - $R_{eq} = \sum_{m=1}^N R_m \dots \dots \dots (6)$

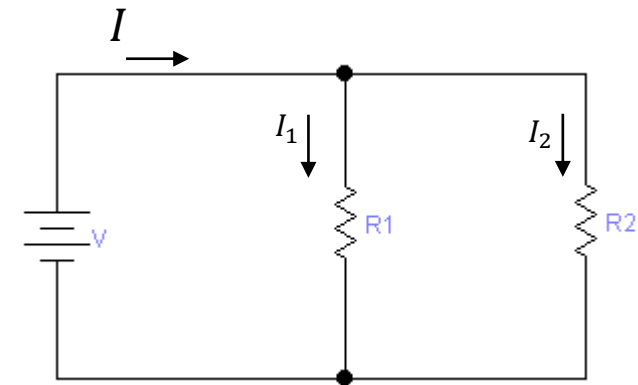


VOLTAGE DIVISION

- When 2 resistors are connected in series the same current flows through both resistors, consequently,
 - $I = \frac{V}{R_1 + R_2} \dots \dots \dots (7)$
- But voltage across each resistor is given by: $V_1 = IR_1$ *and* $V_2 = IR_2$
 - $\therefore V_1 = \left(\frac{V}{R_1 + R_2}\right) R_1 = \left(\frac{R_1}{R_1 + R_2}\right) V \dots \dots \dots (8)$
 - *and* $V_2 = \left(\frac{V}{R_1 + R_2}\right) R_2 = \left(\frac{R_2}{R_1 + R_2}\right) V \dots \dots \dots (9)$
- The ratio of V_1 to V_2 is therefore given as:
 - $\frac{V_1}{V_2} = \frac{R_1}{R_2} \dots \dots \dots (10)$
- If N resistors are in series with a voltage source, the *m*th resistor will have a voltage drop of
 - $V_m = \frac{R_m}{R_1 + R_2 + R_3 + \dots + R_N} V \dots \dots \dots (11)$

Parallel Resistors and Current Division

- From KCL: $I = I_1 + I_2$
- But from Ohm's Law, $I_1 = \frac{V}{R_1}$ and $I_2 = \frac{V}{R_2}$
 - $\therefore I = \frac{V}{R_1} + \frac{V}{R_2} = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$
 - $\therefore \frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$
- But $\frac{I}{V} = \frac{1}{R}$ therefore,
 - $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
 - $\Rightarrow R = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$
- So when two resistors R_1 and R_2 are connected in parallel, they can be replaced by an equivalent resistor R_{eq} where:
 - $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \dots \dots \dots (12)$
 - $\therefore R_{eq} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} \dots \dots \dots (13)$



CURRENT DIVISION

- When 2 resistors are connected in parallel, they have the same voltage across them, consequently,

$$\bullet I_1 = \frac{V}{R_1} = \frac{IR}{R_1} = \frac{I\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}{R_1} = \frac{1}{R_1} \times \frac{R_1 R_2}{R_1 + R_2} \times I = \frac{R_2}{R_1 + R_2} I$$

- Similarly,

$$\bullet I_2 = \frac{V}{R_2} = \frac{IR}{R_2} = \frac{I\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}{R_2} = \frac{1}{R_2} \times \frac{R_1 R_2}{R_1 + R_2} \times I = \frac{R_1}{R_1 + R_2} I$$

- Therefore, If two resistors R_1 and R_2 are connected in parallel with a current source, the current flowing through R_1 and R_2 are:

$$\bullet I_1 = \frac{R_2}{R_1 + R_2} I \dots \dots \dots (14)$$

$$\bullet I_2 = \frac{R_1}{R_1 + R_2} I \dots \dots \dots (15)$$

- The ratio of I_1 to I_2 is therefore given as:

$$\bullet \frac{I_1}{I_2} = \frac{R_2}{R_1} \dots \dots \dots (16)$$

Current division contd.

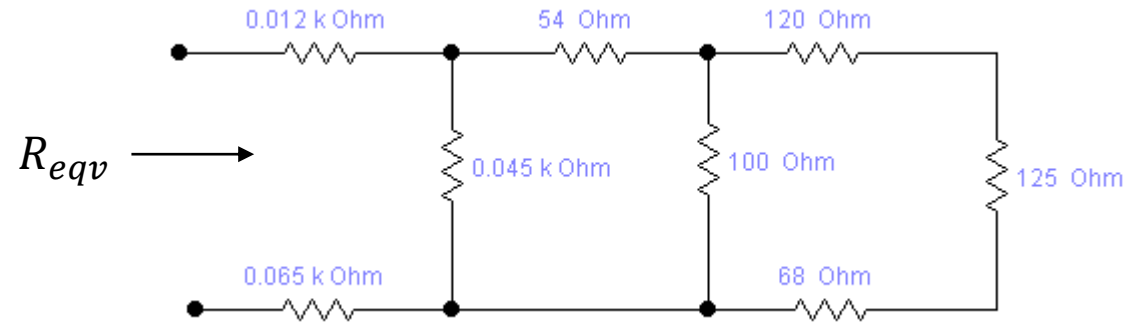
- Eqns 14 and 15 imply that the source current is divided among the resistors in inverse proportion to their resistances. Consequently, it is established that larger current flows through the path of least resistance
- For more than two resistors:

- $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \dots \dots \dots (17)$

- If N resistors are in parallel with a current source, the current flowing through the *m*th resistor can be calculated using the values of the conductance thus:

- $I_m = \frac{G_m}{G_1 + G_2 + G_3 + \dots + G_N} I \dots \dots \dots (18)$

Examples 6: Find R_{eqv} in the network of resistors shown below:



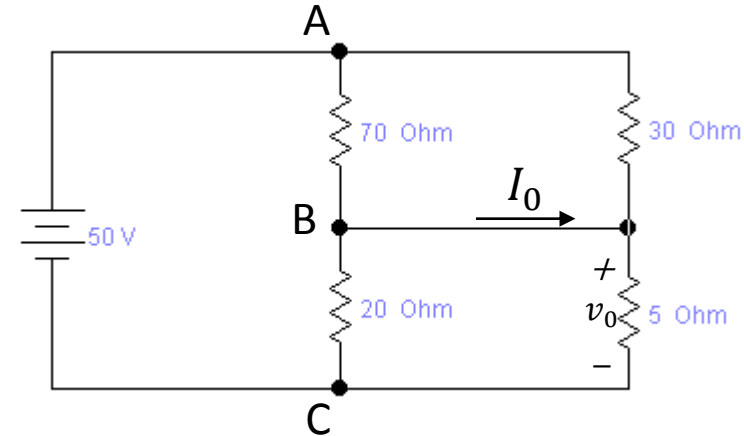
• Solution:

$$• R_{eqv1} = ((125 + 120 + 68) \parallel 100) = \frac{313 \times 100}{313 + 100} = 76 \Omega$$

$$• R_{eqv2} = ((76 + 54) \parallel 45) = \frac{130 \times 45}{130 + 45} = 33 \Omega$$

$$• R_{eqv} = (12 + 33 + 65) = 110 \Omega$$

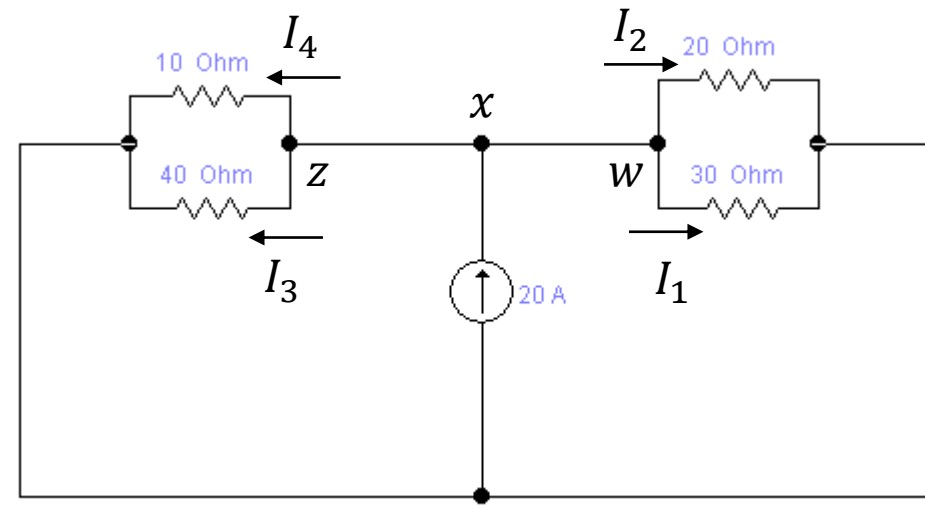
Example 7 : Find v_0 and I_0 in the circuit shown



- Solution
- To find the total Current flowing in the circuit, the total resistance is found thus:
 - $R_{eqv} = (70 \parallel 30) + (20 \parallel 5) = \left(\frac{70 \times 30}{70 + 30}\right) + \left(\frac{20 \times 5}{20 + 5}\right) = 25\Omega$
- From Ohm's law,
 - $I = \frac{V}{R_{eqv}} = \frac{50}{25} = 2A$
- At node A, the current sees two resistors in series: 21Ω and 4Ω , consequently, using VDR:
- Voltage across 21Ω : $v_{21} = \frac{21}{21+4} \times 50 = 42V$
- \therefore since 70Ω and 30Ω are in parallel, they have same $42V$ across them

- Voltage across 4Ω : $v_4 = \frac{4}{21+4} \times 50 = 8V$
 - \therefore since 20Ω and 4Ω are in parallel, they have same $8V$ across them
- To current through the 70Ω : $I_{70} = \frac{42V}{70\Omega} = 0.6A$
- Alternatively, applying CDR, at node A, the $2A$ splits into two,
 - $I_{70} = \frac{30}{70+30} \times 2 = 0.6A$
- At node B, applying KCL:
 - $I_{70} = I_{20} + I_0$
 - But $I_{20} = \frac{8V}{20\Omega} = 0.4A$
 - $\therefore I_0 = I_{70} - I_{20} = 0.6 - 0.4 = 0.2A$
- $I_0 = 0.2A, \quad v_0 = 8V$

Example 8: Find I_1 , I_2 , I_3 , and I_4



- Solution
- Combining the parallel resistors:
 - $R_{10,40} = \frac{10 \times 40}{10 + 40} = 8\Omega$
 - $R_{20,30} = \frac{20 \times 30}{20 + 30} = 12\Omega$
- At node x, the 20A current from the source splits into two according to CDR:
 - $I_{8\Omega} = \frac{12}{8 + 12} \times 20 = 12A$
 - $I_{12\Omega} = \frac{8}{8 + 12} \times 20 = 8A$

- At node w, $I_{12\Omega} = I_1 + I_2$
 - $I_1 = \frac{30}{20 + 30} \times 8 = 4.8A$
 - $I_2 = \frac{20}{20 + 30} \times 8 = 3.2A$
- At node z, $I_{8\Omega} = I_4 + I_3$
 - $I_3 = \frac{10}{10 + 40} \times 12 = 2.4A$
 - $I_4 = \frac{40}{10 + 40} \times 12 = 9.6A$